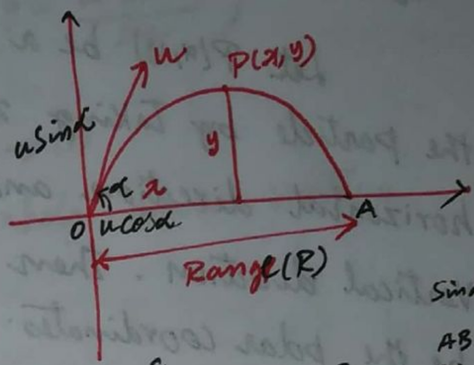
UNIT-2

Projectiles

Projectile: A Particle thrown into air some velocity in any direction is called projectiles.

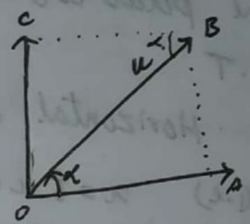
Trajectory: The Path describes by the projectile is called trajectory.

Horizontal Range



The Horizontal Range of a Particle is the distance between the point of Projection 'O' and the point where the curve agin meet the horizontal through 'O'.

It is denoted by 'R'.



$\sin\alpha = \frac{AB}{OB}$

$AB = u\sin\alpha$

$OC = u\sin\alpha$

$\cos\alpha = \frac{OA}{OB}$

$\cos\alpha = \frac{OA}{u}$

$OA = u\cos\alpha$

Note: The Velocity Components along horizontal and Vertical are $u\cos\alpha$ and $u\sin\alpha$ respectively.

Time of flight:

The total Time taken by the particle by the path is called time of flight

Equation of Motion:

i) $V = u + at$

ii) $S = ut + \frac{1}{2}at^2$

iii) $V^2 = u^2 + 2aS$

u – initial Velocity

V – Final Velocity

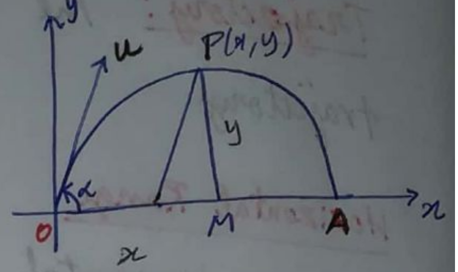
t = Time

a – acceleration, $a \to -g$

S – displacement

1) P.T the Path of the projectile is Parabola and also find the time of flight and Range.

Sol.: Let 'O' be the point of projection and Let 'u' be the velocity of the projection and α be the angle of projection.



Let $P(x,y)$ be a position of the particle by taking x-axis as horizontal direction and y-axis as Vertical direction. Then $P(u\cos\alpha, u\sin\alpha)$ be the polar coordinates.

W.K.T

Horizontal displacement $= t\, u\cos\alpha$

(i.e) $x = t\, u\cos\alpha$

$t = \dfrac{x}{u\cos\alpha}$ $[\because OA = OP,\ x = y$

The initial vertical velocity $= u\sin\alpha$

Acceleration $(a) = -g$

W.K.T $S = ut + \frac{1}{2}at^2$

$y = ut - \frac{1}{2}gt^2$

$y = (u\sin\alpha)\dfrac{x}{u\cos\alpha} - \dfrac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (1)$$

Eqn. (1) is called the equation of trajectory

(x) by $\dfrac{2u^2\cos^2\alpha}{g}$ on both sides

$$\frac{2u^2\cos^2\alpha}{g}\, y = x\frac{\sin\alpha}{\cos\alpha}\cdot\frac{2u^2\cos^2\alpha}{g} - \frac{2u^2\cancel{\cos^2\alpha}}{\cancel{g}}\cdot\frac{\cancel{g}x^2}{2u^2\cancel{\cos^2\alpha}}$$

$$x^2 - \frac{2u^2\cos^2\alpha\tan\alpha}{g}\, x = -\frac{2u^2\cos^2\alpha}{g}\, y$$

$$x^2 - \frac{2u^2\cos^2\alpha\left(\frac{\sin\alpha}{\cos\alpha}\right)x}{g} = -\frac{2u^2\cos^2\alpha}{g}\, y$$

$$x^2 - \frac{2u^2 \cos\alpha \sin\alpha}{g} x = -\frac{2u^2\cos^2\alpha}{g} \cdot y$$

Adding $\frac{u^4 \sin^2\alpha \cdot \cos^2\alpha}{g^2}$ on both sides

$$x^2 - \frac{2u^2 \cos\alpha \sin\alpha}{g} \cdot x + \frac{u^4 \sin^2\alpha \cdot \cos^2\alpha}{g^2} = -\frac{2u^2\cos^2\alpha}{g} y + \frac{u^4 \sin^2\alpha \cdot \cos^2\alpha}{g^2}$$

$$\left(x - \frac{u^2 \sin\alpha \cos\alpha}{g}\right)^2 = \frac{-2u^2\cos^2\alpha}{g}\left(y - \frac{u^2\sin^2\alpha}{2g}\right)$$

$$x^2 = -4ay \longrightarrow (1)$$

Equation (1) represented the parabola whose vertices

$$\left(\frac{u^2 \sin\alpha \cos\alpha}{g}, \frac{u^2\sin^2\alpha}{2g}\right)$$

Length of the latus rectum is

$$4a = \frac{2u^2\cos^2\alpha}{g}$$

Hence the path described by the projection is a parabola

To find the time of flight

Initial vertical velocity is $u\sin\alpha$ and the vertical distance travelled by the particle then the acceleration $a = -g$

W.K.T

$$s = ut + \frac{1}{2}at^2$$

Since $s = 0$,

$$0 = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$2ut\sin\alpha - gt^2 = 0$$

$$t[2u\sin\alpha - gt] = 0$$

$$t \neq 0, \quad 2u\sin\alpha - gt = 0$$

$$2u\sin\alpha = gt$$

$$\boxed{t = \frac{2u\sin\alpha}{g}}$$

To Find the Horizontal Range

W.K.T

$$\text{Range} = \text{distance } OA \times t$$

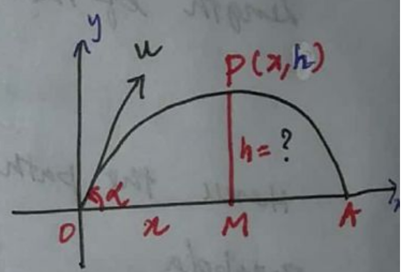
$$= u\cos\alpha \times t$$

$$= u\cos\alpha \cdot \frac{2u\sin\alpha}{g}$$

$$= \frac{2u^2\sin\alpha\cos\alpha}{g}$$

$$\boxed{R = \frac{u^2\sin 2\alpha}{g}}$$

2) Find the maximum height which by the particle and the time taken by the Particle to reach the maximum height.



Sol. Let h denote the maximum height of the particle

$a = -g$

$u = u\sin\alpha$ (initial velocity)

$v = 0$ (terminal velocity)

W.K.T

$v^2 = u^2 + 2aS$

$0 = (u\sin\alpha)^2 - 2gh$

$$\boxed{h = \frac{u^2\sin^2\alpha}{2g}}$$

Let t be the time taken by the Particle to attain maximum height

$v = u + at$

$0 = u\sin\alpha - gt$

$$\boxed{t = \frac{u\sin\alpha}{g}}$$

3) If the greatest height obtained by the Particle is $\frac{1}{4}$ of its horizontal range. Find the angle of projection.

Sol. W.K.T Greatest height $= \frac{u^2 \sin^2\alpha}{2g}$

$$\text{Range} = \frac{u^2 \sin 2\alpha}{g}$$

Given,

$$\text{Greatest height} = \frac{1}{4}\text{ Range}$$

$$\frac{\cancel{u^2}\sin^2\alpha}{\cancel{2g}} = \frac{1}{\cancel{4}_{\,2}}\frac{\cancel{u^2}\sin 2\alpha}{\cancel{g}}$$

$$\sin^{\cancel{2}}\alpha = \frac{1}{\cancel{2}}\cdot\cancel{2}\cancel{\sin}\alpha.\cos\alpha$$

$$\sin\alpha = \cos\alpha \Rightarrow \frac{\sin\alpha}{\cos\alpha} = 1 \Rightarrow \tan\alpha = 1$$

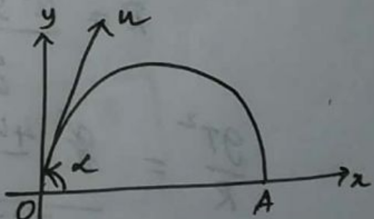
$$\alpha = \tan^{-1}(1)$$

$$\boxed{\alpha = \pi/4}$$

4) S.T the greatest height obtained by the particle with the velocity u at an angle α with the horizontal is unaltered. If the velocity of projection is increased to Ku and the angle of projection is decreased by λ then $\text{Cosec}\,\lambda = K(\cot\lambda - \cot\alpha)$.

Sol. W.K.T

Greatest height $(H) = \frac{u^2 \sin^2\alpha}{2g}$



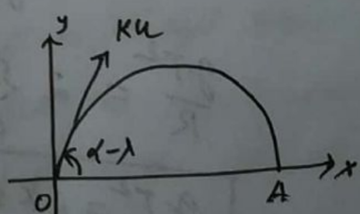
Given

$u \rightarrow Ku$

$\alpha \rightarrow \alpha - \lambda$

then

Greatest height $(H_1) = \frac{(Ku)^2(\sin(\alpha-\lambda))^2}{2g}$



Given $H = H_1$

$$\frac{u^2\sin^2\alpha}{2g} = \frac{K^2u^2(\sin(\alpha-\lambda))^2}{2g}$$

$$\sin^2\alpha = K^2(\sin\alpha\cos\lambda - \cos\alpha\sin\lambda)^2$$

Taking Square root on both sides

$$\sin\alpha = K(\sin\alpha\cos\lambda - \cos\alpha\sin\lambda)$$

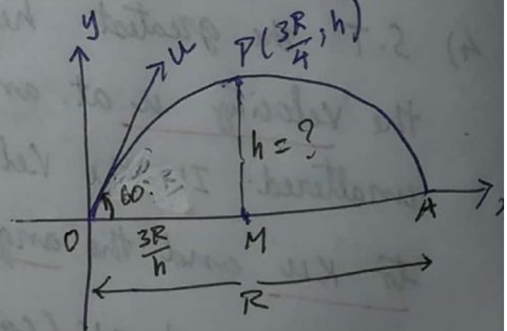
$\div$ by $\sin\alpha$, $1 = K\left(\cos\lambda - \frac{\cos\alpha}{\sin\alpha}.\sin\lambda\right)$

$\div$ by $\sin\lambda$, $\frac{1}{\sin\lambda} = K\left(\frac{\cos\lambda}{\sin\lambda} - \cot\alpha.\frac{\sin\lambda}{\sin\lambda}\right)$

$$\boxed{\operatorname{cosec}\lambda = K(\cot\lambda - \cot\alpha)}$$

—x—

5) If T is the time of flight and R is the Horizontal Range P.T (i) $gT^2 = 2R\tan\alpha$. Where α is the angle of projection (ii) If α is $60°$, Find height interms of R. Find the height of the projectile above the point of projection when its move to the horizontal distance $\frac{3R}{4}$.



Sol. 1) $gT^2 = 2R\tan\alpha$

$$\frac{gT^2}{R} = 2\tan\alpha$$

W.K.T

$$T = \frac{2u\sin\alpha}{g}$$

$$R = \frac{u^2\sin 2\alpha}{g}$$

$$\frac{gT^2}{R} = \frac{\cancel{g}\,\frac{4u^2\sin^2\alpha}{g^{\cancel{2}}}}{\frac{u^2\sin 2\alpha}{g}} = \frac{\cancel{4}^{2}\cancel{u^2}\sin^{\cancel{2}}\alpha}{g} \times \frac{g}{\cancel{u^2}\cancel{2}\cancel{\sin\alpha}.\cos\alpha}$$

$$\frac{gT^2}{R} = 2\,\frac{\sin\alpha}{\cos\alpha}$$

$$\boxed{gT^2 = 2R\tan\alpha}$$

Hence (i) proved

ii) Angle $(\alpha) = 60°$, $h = ?$

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$\sin 2x = 2 \sin x \cos x$

$\sin(\pi/2 + \theta) = \cos\theta$

$$= \frac{u^2 \sin 2(60)}{g}$$

$$= \frac{u^2 \sin(90+30)}{g} = \frac{u^2 \cos 30}{g}$$

$$R = \frac{\sqrt{3} u^2}{2g}$$

$$u^2 = \frac{2Rg}{\sqrt{3}}$$

Equation trajectory is

$$y = x \tan\alpha - \frac{g x^2}{2u^2 \cos^2\alpha}$$

$P(\frac{3R}{4}, h)$ lies on trajectory

$$h = \frac{3R}{4} \tan 60° - \frac{g\left(\frac{3R}{4}\right)^2}{2\left(\frac{2Rg}{\sqrt{3}}\right)\cos^2 60}$$

$$= \frac{3R\sqrt{3}}{4} - \frac{g(9R^2)}{16 \times 4Rg\left(\frac{1}{4}\right)} \cdot \sqrt{3}$$

$$= \frac{3\sqrt{3}R}{4} - \frac{9\sqrt{3}R}{16}$$

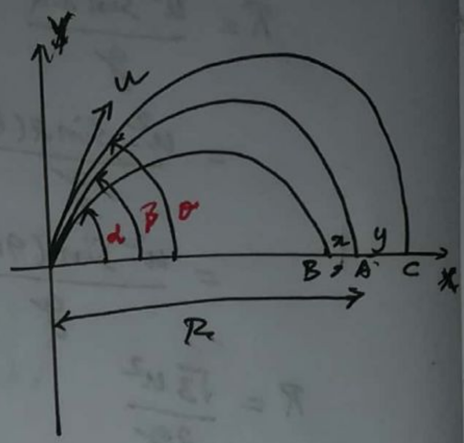
$$= \frac{3\sqrt{3}R}{4}\left(1 - \frac{3}{4}\right)$$

$$= \frac{3}{4}\sqrt{3} R \left(\frac{1}{4}\right) \Rightarrow \boxed{h = \frac{3\sqrt{3}R}{16}}$$

6) Two Particle are projected from the Same point 'O' with Same Velocity at angle α and β aim at a target on the horizontal through 'O'. One falls x feet to short and other falls y feet for from to target. If θ be the current angle of the projection. so as to hit target Show that $(x+y)\sin 2\theta = x \sin 2\beta + y \sin 2\alpha$.

Sol.: Let velocity of Projection is u

Let θ be the angle of Projection



$$OA = \text{Range} = R$$

$$OA = \frac{u^2 \sin 2\theta}{g}$$

Given $AB = x$, $AC = y$

1) Velocity $= u$

Angle of projection $= \alpha$

$$OB = \frac{u^2 \sin 2\alpha}{g}$$

$$OA - AB = \frac{u^2 \sin 2\alpha}{g}$$

$$R - x = \frac{u^2 \sin 2\alpha}{g}$$

$$\frac{u^2 \sin 2\theta}{g} - x = \frac{u^2 \sin 2\alpha}{g} \longrightarrow (1)$$

ii) Velocity $= u$

Angle of projection $= \beta$

$$OC = \frac{u^2 \sin 2\beta}{g}$$

$$OA + AC = \frac{u^2 \sin 2\beta}{g}$$

$$R + y = \frac{u^2 \sin 2\beta}{g}$$

$$\frac{u^2 \sin 2\theta}{g} + y = \frac{u^2 \sin 2\beta}{g} \longrightarrow (2)$$

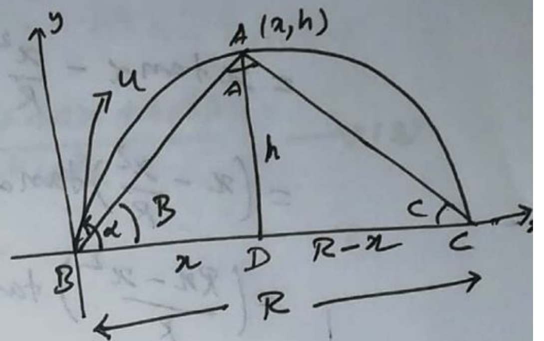
$$(1) \times y \Rightarrow \quad y\frac{u^2 \sin 2\theta}{g} - \cancel{xy} = y\frac{u^2 \sin 2\alpha}{g}$$

$$(2) \times x \Rightarrow \quad x\frac{u^2 \sin 2\theta}{g} + \cancel{xy} = x\frac{u^2 \sin 2\beta}{g}$$

$$(x+y)\frac{\cancel{u^2} \sin 2\theta}{\cancel{g}} = \frac{\cancel{u^2}}{\cancel{g}}(x \sin 2\beta + y \sin 2\alpha)$$

$$(x+y) \sin 2\theta = x \sin 2\beta + y \sin 2\alpha$$

7) A Particle is projected over a triangle from one end of its horizontal base to reach the vertex at the other end of the base. If B and C base and α be the angle of projection the prove that $\tan\alpha = \tan B + \tan C$.



Sol. Let ABC be triangle

u be a projection of velocity

α angle of projection

ΔBAD,

$$\tan B = \frac{h}{x}$$

ΔADC,

$$\tan C = \frac{h}{R-x}$$

$$\tan B + \tan C = \frac{h}{x} + \frac{h}{R-x}$$

$$= \frac{(R-\cancel{x}+\cancel{x})h}{x(R-x)}$$

$$\tan B + \tan C = h\left(\frac{R}{x(R-x)}\right) \longrightarrow (1)$$

Equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (2)$$

$P(x,y)$ lies on (2)

$$h = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (3)$$

W.K.T

$$R = \frac{u^2\sin 2\alpha}{g}$$

$$u^2 = \frac{Rg}{\sin 2\alpha}$$

$$(3) \Rightarrow h = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$= x\tan\alpha - \frac{gx^2}{2\times\frac{Rg}{\sin 2\alpha}\cos^2\alpha}$$

$$= x\tan\alpha - \frac{x^2 \sin 2\alpha}{2R\cos^2\alpha}$$

$$= x\tan\alpha - \frac{x^2 \cancel{2}\sin\alpha \cancel{\cos\alpha}}{\cancel{2}R\cos^{\cancel{2}}\alpha}$$

$$= x\tan\alpha - \frac{x^2}{R}\tan\alpha$$

$$= \left(x - \frac{x^2}{R}\right)\tan\alpha$$

$$= \left(\frac{Rx - x^2}{R}\right)\tan\alpha$$

$$h = \left(\frac{R-x}{R}\right) x\tan\alpha$$

$$\frac{Rh}{x(R-x)} = \tan\alpha \longrightarrow (4)$$

from (1) & (4) we get

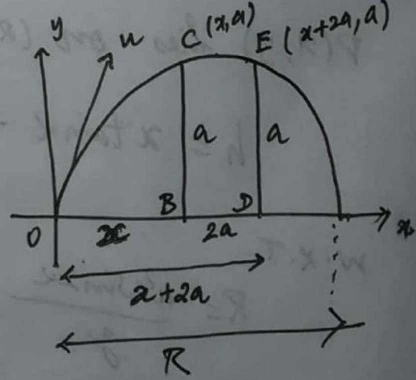
$$\tan B + \tan C = \frac{hR}{x(R-x)} = \tan\alpha$$

$$\therefore \boxed{\tan\alpha = \tan B + \tan C}$$

8) A Particle is projected at an angle α to the horizontal so as to clear two walls of height 'a' at a distance '2a' apart Show that the horizontal Range is $2a\cot\alpha/2$.

Sol.: Let u - velocity of Projection

α - angle of Projection



$BC = DE = a$

$BD = 2a$

$OA = R$

The coordinates of C is (x, a) and the coordinates of E is $(x+2a, a)$

Equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (1)$$

$C(x, a)$ lies on (1)

$$a = x \tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (2)$$

$E(x+2a, a)$ lies on (1)

$$a = (x+2a)\tan\alpha - \frac{g(x+2a)^2}{2u^2\cos^2\alpha}$$

$$a = x\tan\alpha + 2a\tan\alpha - \frac{g(x^2+4ax+4a^2)}{2u^2\cos^2\alpha} \longrightarrow (3)$$

$(2)-(3) \Rightarrow$

$$0 = -2a\tan\alpha + \frac{(4ax+4a^2)g}{2u^2\cos^2\alpha}$$

$$0 = -2a\tan\alpha + \frac{4ag(x+a)}{2u^2\cos^2\alpha}$$

$$2a\tan\alpha\,(2u^2\cos^2\alpha) = 4ag(x+a)$$

$$\cancel{2a}\,\frac{\sin\alpha}{\cos\alpha}(\cancel{2}u^2\cos^2\alpha) = \cancel{4a}\,g(x+a)$$

$$\sin\alpha\cos\alpha\, u^2 = g(x+a)$$

$$\cancel{\sin\alpha}\,\cancel{\cos\alpha}\cdot\frac{R\cancel{g}}{2\cancel{\sin\alpha}\,\cancel{\cos\alpha}} = \cancel{g}(x+a)$$

$$\frac{R}{2} = x+a \Rightarrow \frac{R-a}{2} = x$$

$$\boxed{x = \frac{R-2a}{2}} \longrightarrow (3)$$

Sub. the value of (3) in (2)

$$a = \left(\frac{R-2a}{2}\right)\tan\alpha - \frac{g\left(\frac{R-2a}{2}\right)^2}{2\cos^2\alpha\left(\frac{Rg}{2\sin\alpha\cos\alpha}\right)}$$

$$a = \left(\frac{R-2a}{2}\right)\tan\alpha - \frac{(R-2a)^2}{4R\cot\alpha}$$

$$a = \left(\frac{R-2a}{2}\right)\tan\alpha - \frac{(R-2a)^2}{4R}\tan\alpha$$

$$4Ra = 2R(R-2a)\tan\alpha - (R-2a)^2\tan\alpha$$

$$= (R-2a)\tan\alpha\,(2R-(R-2a))$$

$$= (R-2a)\tan\alpha\,(R+2a)$$

$$= (R^2+4a^2)\tan\alpha$$

$$4Ra = R^2\tan\alpha + 4a^2\tan\alpha$$

$$R^2\tan\alpha - 4Ra - 4a^2\tan\alpha = 0$$

$$A = \tan\alpha, \quad B = -4a, \quad C = -4a^2\tan\alpha$$

$$R = \frac{4a \pm \sqrt{16a^2 + 16a^2\tan^2\alpha}}{2\tan\alpha}$$

$$= \frac{4a \pm 4a\sqrt{1+\tan^2\alpha}}{2\tan\alpha}$$

$$= \frac{4a \pm 4a\sqrt{\sec^2\alpha}}{2\tan\alpha}$$

$$= \frac{\cancel{4}a(1\pm\sec\alpha)}{\cancel{2}\tan\alpha} = \frac{1+\sec\alpha}{\tan\alpha} \quad (\text{or}) \quad \frac{1-\sec\alpha}{\tan\alpha}$$

$$= \frac{2a\left(1 + \frac{1}{\cos\alpha}\right)}{\frac{\sin\alpha}{\cos\alpha}}$$

$$= \frac{2a\left(\frac{\cos\alpha+1}{\cancel{\cos\alpha}}\right)}{\frac{\sin\alpha}{\cancel{\cos\alpha}}}$$

$$= \frac{2a(1+\cos\alpha)}{\sin\alpha}$$

$\because \cos^2\alpha = \frac{1+\cos2\alpha}{2}$

$\sin2\alpha = 2\sin\alpha\cos\alpha$

$$= \frac{\cancel{2}a(2\cos^2\alpha/2)}{\cancel{2}\sin\alpha/2\,\cancel{\cos\alpha/2}}$$

$$= 2a\frac{\cos\alpha/2}{\sin\alpha/2}$$

$$\boxed{R = 2a\cot\alpha/2}$$

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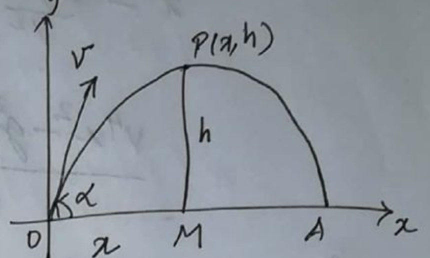
Q) A Particle is projected from the fixed point on the ground level show that it cannot clear the wall of height 'h' at a distance x from the point of projection is $v^2 < g[\sqrt{x^2+h^2}+h]$

Sol.



Velocity of projection $= v$

Angle of projection $= \alpha$

$PM = h$, $OM = x$

coordinates of P is (x, h)

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (1)$$

$P(x, h)$ lies on (1)

$$h = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha}$$

$$h = x\tan\alpha - \frac{gx^2}{2v^2}\sec^2\alpha$$

$$h = x\tan\alpha - \frac{gx^2}{2v^2}(1+\tan^2\alpha)$$

$$h = x\tan\alpha - \frac{gx^2}{2v^2} - \frac{gx^2}{2v^2}\tan^2\alpha$$

$$\frac{gx^2}{2v^2}\tan^2\alpha - x\tan\alpha + \left(\frac{gx^2}{2v^2}+h\right) = 0$$

$\tan\alpha = t$,

$$\frac{gx^2t^2}{2v^2} - xt + \left(\frac{gx^2}{2v^2}+h\right) = 0$$

t is real and distinct

$$b^2 - 4ac \geq 0$$

where $a = \frac{gx^2}{2v^2}$, $b = -x$, $c = \frac{gx^2}{2v^2}+h$

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$$x^2 - 4\left(\frac{gx^2}{2v^2}\right)\left(\frac{gx^2}{2v^2}+h\right) \geq 0$$

$$x^2 - \frac{2gx^2}{v^2}\left(\frac{gx^2}{2v^2}+h\right) \geq 0$$

$$x^2 - \frac{2gx^2}{v^2}\cdot\frac{gx^2}{2v^2} - \frac{2gx^2}{v^2}\cdot h \geq 0$$

$$\frac{v^4x^2 - g^2x^4 - 2gv^2x^2h}{v^4} \geq 0$$

$$v^4x^2 - g^2x^4 - 2gv^2x^2h \geq 0$$

$\div x^2$,

$$v^4 - g^2x^2 - 2gv^2h \geq 0$$

$$v^4 - 2gv^2h \geq g^2x^2$$

$$(v^2)^2 - 2ghv^2 + (gh)^2 \geq g^2x^2 + (gh)^2$$

$$(v^2 - gh)^2 \geq g^2(x^2+h^2)$$

$$v^2 - gh \geq \sqrt{g^2(x^2+h^2)}$$

$$v^2 \geq gh + g\sqrt{x^2+h^2}$$

$$v^2 \geq g(h + \sqrt{x^2+h^2})$$

If the particle is clear the wall then

$$v^2 \geq g(h+\sqrt{x^2+h^2})$$

If the particle is cannot clear the wall then

$$v^2 < g(h+\sqrt{x^2+h^2})$$

—x—

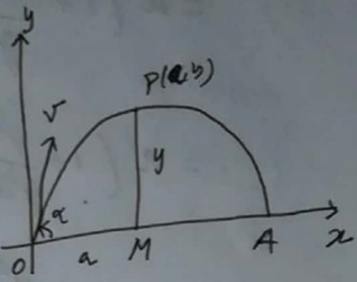
10) S.T the greatest height which a particle withe an initial velocity 'v' can reach a vertical wall at a distance 'a' from the point of projection is

$$y = \frac{v^2}{2g} - \frac{ga^2}{2v^2}$$

Soln.

Let $V \rightarrow$ velocity of projection

$\alpha \rightarrow$ Angle of projection



$OM = a, \quad PM = y$

The coordinates of $P(a, y)$

Equation of trajectory

$$y = x \tan\alpha - \frac{g x^2}{2V^2 \cos^2\alpha} \longrightarrow (1)$$

$P(a, y)$ lies on (1).

$$y = a \tan\alpha - \frac{g a^2}{2V^2} \sec^2\alpha \longrightarrow (\cancel{2})$$

Diff. w.r.t 'α',

$$\frac{dy}{d\alpha} = a \sec^2\alpha - \frac{g a^2}{2V^2} 2\sec\alpha \sec\alpha \tan\alpha$$

$$= a\sec^2\alpha - \frac{g a^2}{2V^2} 2\sec^2\alpha \tan\alpha$$

$$= a \sec^2\alpha \left(1 - \frac{g a}{V^2} \tan\alpha \right) \longrightarrow (2)$$

$\frac{dy}{d\alpha} = 0,$

$$a \sec^2\alpha \left(1 - \frac{g a}{V^2} \tan\alpha\right) = 0$$

$$a \sec^2\alpha \neq 0, \quad 1 - \frac{g a}{V^2} \tan\alpha = 0$$

$$\boxed{\tan\alpha = \frac{V^2}{g a}} \longrightarrow (3)$$

Diff. (3) w.r.t 'α',

$$\frac{d^2 y}{d\alpha^2} = a \cdot 2\sec\alpha(\sec\alpha . \tan\alpha) - \frac{g a^2}{V^2}(\sec^2\alpha \sec^2\alpha + \tan\alpha . 2\sec^2\alpha \tan\alpha)$$

$$= 2a \sec^2\alpha \tan\alpha - \frac{g a^2}{V^2}(\sec^4\alpha + 2\sec^2\alpha \tan^2\alpha)$$

$$= a\sec^2\alpha\left[2\tan\alpha - \frac{g a}{V^2}(\sec^2\alpha + 2\tan^2\alpha)\right]$$

$$= a(1 + \tan^2\alpha)\left[2\tan\alpha - \frac{g a}{V^2}(1 + \tan^2\alpha + 2\tan^2\alpha)\right]$$

$$= a(1 + \tan^2\alpha)\left[2\tan\alpha - \frac{g a}{V^2}(1 + 3\tan^2\alpha)\right]$$

$$= a\left(1+\frac{v^4}{g^2a^2}\right)\left[\frac{2v^2}{ga} - \frac{ga}{v^2}\left(1+\frac{3v^4}{g^2a^2}\right)\right]$$

$$= a\left(1+\frac{v^4}{g^2a^2}\right)\left(\frac{2v^2}{ga} - \frac{ga}{v^2} - \frac{\cancel{ga}}{\cancel{v^2}}\cdot\frac{3v^{4^{2}}}{g^{\cancel{2}}a^{\cancel{2}}}\right)$$

$$= a\left(1+\frac{v^4}{g^2a^2}\right)\left(\frac{2v^2}{ga} - \frac{ga}{v^2} - \frac{3v^2}{ga}\right)$$

$$= a\left(1+\frac{v^4}{g^2a^2}\right)\left(\frac{-v^2}{ga} - \frac{ga}{v^2}\right)$$

$$\frac{d^2y}{d\alpha^2} = -a\left(1+\frac{v^4}{g^2a^2}\right)\left(\frac{v^2}{ga} + \frac{ga}{v^2}\right)$$

$$\frac{d^2y}{d\alpha^2} < 0$$

$\tan\alpha = \frac{v^2}{ga}$ attains the maximum height

Sub. (3) in (*)

$$y = \cancel{a}\left(\frac{v^2}{g\cancel{a}}\right) - \frac{ga^2}{2v^2}\left(1+\frac{v^4}{g^2a^2}\right)$$

$$= \frac{v^2}{g} - \frac{ga^2}{2v^2} - \frac{v^2}{2g}$$

$$= \frac{2v^2 - v^2}{2g} - \frac{ga^2}{2v^2}$$

$$\boxed{y = \frac{v^2}{2g} - \frac{ga^2}{2v^2}}$$

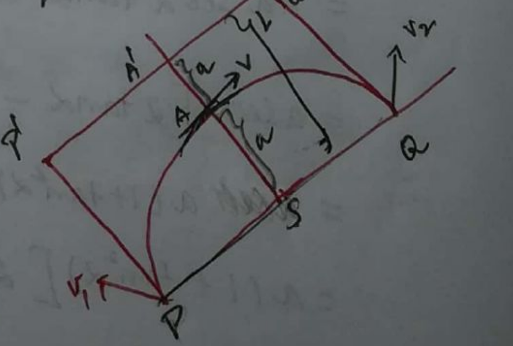
11) V_1 and V_2 are velocities of a projectile at the ends of focal chords of the path of the projectile and V is the horizontal velocity. P.T. $\frac{1}{V_1^2} + \frac{1}{V_2^2} = \frac{1}{V^2}$ (or) $V_1^{-2} + V_2^{-2} = V^{-2}$

Sol.

$$\frac{SP}{PP'} = 1$$

$$SP = PP'$$

$$SA = AA'$$



$$\frac{SQ}{QQ'} = 1$$

$$AA' = a = SA$$

$$2AA' = 2a = L$$

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{L} \longrightarrow (1)$$

At PP'

$$V^2 = u^2 + 2as$$

$$V_1^2 = 0 + 2g\,PP'$$

$$PP' = \frac{V_1^2}{2g}$$

At QQ'

$$V^2 = u^2 + 2as$$

$$V_2^2 = 0 + 2g\,QQ'$$

$$QQ' = \frac{V_2^2}{2g}$$

At AA'

$$V^2 = u^2 + 2as$$

$$V^2 = 0 + 2g\,AA'$$

$$AA' = \frac{V^2}{2g}$$

from (1)

$$\frac{1}{PP'} + \frac{1}{QQ'} = \frac{2}{2AA'}$$

$$\frac{1}{V_1^2/2g} + \frac{1}{V_2^2/2g} = \frac{\cancel{2}}{\cancel{2}\left(\frac{V^2}{2g}\right)}$$

$$\frac{2g}{V_1^2} + \frac{2g}{V_2^2} = \frac{2g}{V^2}$$

$\div$ by $2g$, $\frac{1}{V_1^2} + \frac{1}{V_2^2} = \frac{1}{V^2}$

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12) Two particles are projected into the two different directions with same speed, so that they have equal horizontal range, if the greatest height attained by them all h_1, h_2 then S.T Range $R = 4\sqrt{h_1 h_2}$.

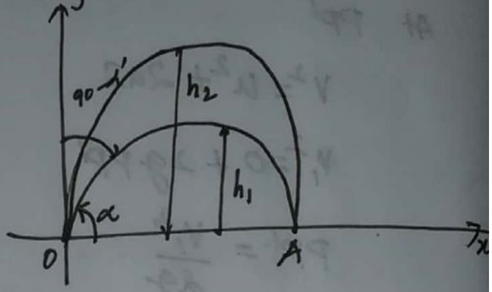
Sol:

Let

u - Velocity of projection

α - Angle of projection

h_1 - the maximum height



$$R = \frac{u^2 \sin 2\alpha}{g}$$

$$\boxed{h_1 = \frac{u^2 \sin^2 \alpha}{2g}}$$

Let u - Velocity of projection

$90° - \alpha$ - Angle of projection

h_2 - the maximum height

$$h_2 = \frac{u^2 \sin^2(90 - \alpha)}{2g}$$

$$\boxed{h_2 = \frac{u^2 \cos^2 \alpha}{2g}}$$

$$h_1 h_2 = \frac{u^2 \sin^2 \alpha}{2g} \cdot \frac{u^2 \cos^2 \alpha}{2g}$$

$$h_1 h_2 = \frac{u^4 \sin^2 \alpha \cdot \cos^2 \alpha}{4g^2}$$

$$\sqrt{h_1 h_2} = \frac{u^2 \sin \alpha \cos \alpha}{2g}$$

$$= \frac{u^2 \frac{\sin 2\alpha}{2}}{2g} = \frac{u^2 \sin 2\alpha}{4g} \Rightarrow 4\sqrt{h_1 h_2} = \frac{u^2 \sin 2\alpha}{g}$$

$$\boxed{4\sqrt{h_1 h_2} = R}$$

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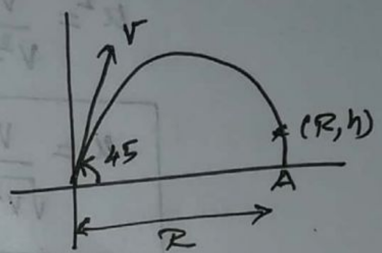
13) A Shot of projectile with velocity 'V' at an angle of inclination is 45° and reaches the point a through the point of projection. S.T to hit a mark at a height 'h' above A projection the Start at the Same elevation the velocity of projection must be increased to

$$\frac{V^2}{\sqrt{V^2-hg}}.$$

Sol.

Let V be a velocity of projection and 45° - angle of projection.



$$OA = R = \frac{u^2 \sin 2\alpha}{g}$$

$$= \frac{V^2 \sin 2\alpha}{g}$$

$$= \frac{V^2 \sin 90}{g}$$

$$R = \frac{V^2}{g}$$

Equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (1)$$

(R, h) lies in (1)

$$h = R\tan 45° - \frac{gR^2}{2u^2\cos^2 45}$$

$$h = R - \frac{gR^2}{u^2}$$

Sub. the value $R = \frac{V^2}{g}$

$$h = \frac{V^2}{g} - \frac{g\left(\frac{V^2}{g}\right)^2}{u^2} = \frac{V^2}{g} - \frac{\cancel{g}\left(\frac{V^4}{g^{\cancel{2}}}\right)}{u^2}$$

$$h = \frac{V^2}{g} - \frac{V^4}{gu^2}$$

$$h = \frac{u^2v^2 - v^4}{gu^2}$$

$$hgu^2 = u^2v^2 - v^4$$

$$hgu^2 - u^2v^2 = -v^4$$

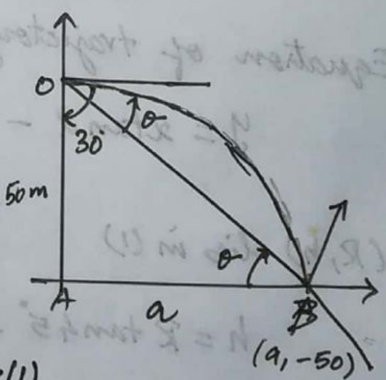
$$u^2(hg - v^2) = -v^4$$

$$u^2 = \frac{-v^4}{hg - v^2}$$

$$u^2 = \frac{v^4}{v^2 - hg}$$

$$\boxed{u = \frac{v^2}{\sqrt{v^2 - hg}}}$$

14) A Particle projected from the top 'O' of a wall AO, 50 m height at an angle of 30° above the horizontal strikes the level ground through 'A' at 'B' at an angle of 45°. S.T the angle depression of B from O is $\tan^{-1}\frac{\sqrt{3}-1}{2\sqrt{3}}$.



Sol.

Let O - point of projection

45° - angle of projection

Equation of trajectory

$$y = \tan x - \frac{gx^2}{2u^2\cos\alpha} \longrightarrow (1)$$

Let AB = a, the Coordinate B is (a, -50) below the origin B(a, -50) pass through (1)

$$-50 = a\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha} \longrightarrow (2)$$

Slope $= \tan 135$

$= \tan(90° + 45°)$

$$= -\cot 45$$

$$\frac{dy}{dx} = -1$$

Diff. w.r.t 'x', in eqn. (1)

$$\frac{dy}{dx} = \tan\alpha - \frac{2gx}{2u^2\cos^2\alpha}$$

$$= \tan\alpha - \frac{gx}{u^2\cos^2\alpha} \qquad (\because x = a)$$

$$-1 = \tan\alpha - \frac{ga}{u^2\cos^2\alpha} \longrightarrow (3)$$

$$(2) \Rightarrow \quad -50 = a\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha}$$

$$(3)\times\frac{a}{2} \Rightarrow \quad -\frac{a}{2} = \frac{a}{2}\tan\alpha - \frac{ga^2}{2u^2\cos^2\alpha}$$

$$+ \qquad - \qquad +$$

$$\frac{a}{2} - 50 = \frac{a}{2}\tan\alpha$$

$$\frac{a}{2} - 50 = \frac{a}{2}\tan 30$$

$$\frac{a}{2} - 50 = \frac{a}{2}\cdot\frac{1}{\sqrt{3}}$$

$$\frac{a}{2} - \frac{a}{2}\frac{1}{\sqrt{3}} = 50$$

$$\frac{a}{2}\left(1 - \frac{1}{\sqrt{3}}\right) = 50 \quad \Rightarrow \quad \frac{a}{2}\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 50$$

$$a = \frac{100\sqrt{3}}{\sqrt{3}-1} \qquad (= AB)$$

ΔOAB;

$$\tan\theta = \frac{OA}{AB}$$

$$= \frac{\cancel{50}}{(1\cancel{00})\frac{\sqrt{3}}{\sqrt{3}-1}} = \frac{\sqrt{3}-1}{2\sqrt{3}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right)}$$

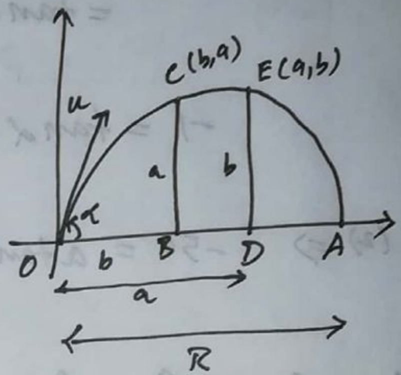
15) A ball is projected so as to clear the two walls first of height 'a' at a distance 'b' from the point of projection and the second of height 'b' at a distance 'a' from the point of projection. S.T $R = \frac{a^2+ab+b^2}{a+b}$ and angle of projection exceeds $\tan^{-1}(3)$.

Sol.

Let O - point of projection

α - angle of projection

u - velocity



$OB = b$, $BC = a$

Coordinates of $C(b,a)$

$OD = a$, $DE = b$

Coordinates of $E(a,b)$

Equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (1)$$

$C(b,a)$ lies on (1)

$$a = b\tan\alpha - \frac{gb^2}{2u^2\cos^2\alpha} \longrightarrow (2)$$

$E(a,b)$ lies on (1)

$$b = a\tan\alpha - \frac{a^2 g}{2u^2\cos^2\alpha} \longrightarrow (3)$$

$$(2)\times b \Rightarrow \cancel{ab} = b^2\tan\alpha - \frac{gb^3}{2u^2\cos^2\alpha}$$

$$(3)\times a \Rightarrow \cancel{ab} = a^2\tan\alpha - \frac{ga^3}{2u^2\cos^2\alpha}$$

$$\quad - \quad\quad - \quad\quad +$$

$$0 = (b^2-a^2)\tan\alpha + \frac{g}{2u^2\cos^2\alpha}(a^3-b^3)$$

$$0 = -(a^2-b^2)\tan\alpha + \frac{g}{2u^2\cos^2\alpha}(a^3-b^3)$$

$$\cancel{(a-b)}(a+b)\frac{\sin\alpha}{\cos\alpha} = \frac{g}{2u^2\cos^2\alpha}\cancel{(a-b)}(a^2+ab+b^2)$$

$$\frac{2u^2\sin\alpha\cos\alpha}{g} = \frac{a^2+ab+b^2}{a+b}$$

$$\frac{u^2\sin 2\alpha}{g} = \frac{a^2+ab+b^2}{a+b}$$

$$R = \frac{a^2+ab+b^2}{a+b}$$

$$(2) \Rightarrow \quad a - b\tan\alpha = \frac{-gb^2}{2u^2\cos^2\alpha} \longrightarrow (4)$$

$$(3) \Rightarrow \quad b - a\tan\alpha = \frac{-ga^2}{2u^2\cos^2\alpha} \longrightarrow (5)$$

$$\frac{(4)}{(5)} \Rightarrow \quad \frac{a-b\tan\alpha}{b-a\tan\alpha} = \frac{-\cancel{g}b^2}{\cancel{2u^2\cos^2\alpha}} \times \frac{-\cancel{2u^2\cos^2\alpha}}{\cancel{g}a^2}$$

$$\frac{a-b\tan\alpha}{b-a\tan\alpha} = \frac{b^2}{a^2}$$

$$a^2(a-b\tan\alpha) = b^2(b-a\tan\alpha)$$

$$a^3 - a^2b\tan\alpha = b^3 - ab^2\tan\alpha$$

$$ab^2\tan\alpha - a^2b\tan\alpha = b^3 - a^3$$

$$ab\tan\alpha\,\cancel{(b-a)} = \cancel{(b-a)}(b^2+ab+a^2)$$

$$ab\tan\alpha = a^2+b^2+ab > 2ab+ab$$

$$\because (a-b)^2 > 0$$
$$a^2+b^2-2ab > 0$$
$$a^2+b^2 = 2ab$$

$$ab\tan\alpha > 3ab$$

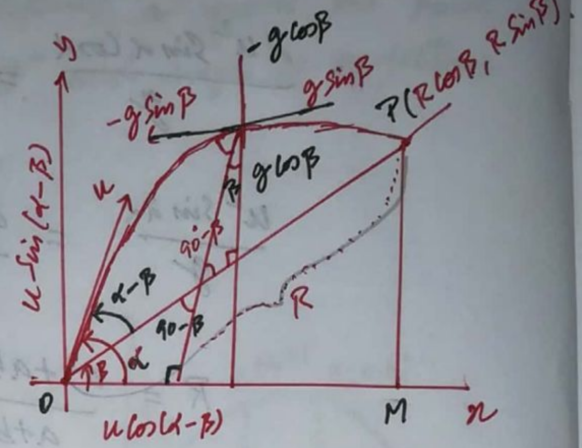
$$\tan\alpha > \frac{3\cancel{ab}}{\cancel{ab}}$$

$$\tan\alpha > 3$$

$$\boxed{\alpha > \tan^{-1}(3)}$$

Particle Projected on an inclined plane:

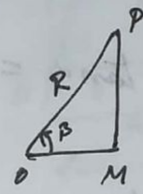
Book work: When a particle is projected from a point 'O' on a plane inclination of 'β' with velocity 'u' making an angle 'α' with the horizontal to find



(i) The Range on the inclined plane

ii) Time of flight

Sol.:



$\sin\beta = \frac{PM}{OP}$

$\boxed{PM = R\sin\beta}$

$\cos\beta = \frac{OM}{OP} = \frac{OM}{R}$

$\boxed{OM = R\cos\beta}$

i) Let u be the velocity of projection angle of projection is $(\alpha-\beta)$

Range $OP = R$

The velocity components are $u\cos(\alpha-\beta)$ and $u\sin(\alpha-\beta)$

$\therefore$ coordinates of P is $(R\cos\beta, R\sin\beta)$

Equation of trajectory

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha} \longrightarrow (1)$$

$P(R\cos\beta, R\sin\beta)$ lies on (1)

$$R\sin\beta = R\cos\beta\tan\alpha - \frac{gR^2\cos^2\beta}{2u^2\cos^2\alpha}$$

$$R\sin\beta = R\cos\beta\frac{\sin\alpha}{\cos\alpha} - \frac{gR^2\cos^2\beta}{2u^2\cos^2\alpha}$$

$$R\sin\beta - R\cos\beta\frac{\sin\alpha}{\cos\alpha} = -\frac{gR^2\cos^2\beta}{2u^2\cos^2\alpha}$$

$$\frac{\cancel{R}(\sin\beta\cos\alpha - \cos\beta\sin\alpha)}{\cos\alpha} = -\frac{gR^{\cancel{2}}\cos^2\beta}{2u^2\cos^2\alpha}$$

$$\frac{(\sin\alpha\cos\beta - \sin\beta\cos\alpha)}{\cos\alpha} = \frac{gR\cos^2\beta}{2u^2\cos^{\cancel{2}}\alpha}$$

$$\sin(\alpha-\beta) = \frac{gR\cos^2\beta}{2u^2\cos\alpha}$$

$$\sin(\alpha-\beta)\, 2u^2\cos\alpha = gR\cos^2\beta$$

$$R = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

ii) Time of flight on inclined plane

W.K.T

$u = u\sin(\alpha-\beta)$

$a = -g\cos\beta$

$$S = ut + \frac{1}{2}at^2$$

$$0 = ut\sin(\alpha-\beta) + \frac{1}{2}t^2(-g\cos\beta)$$

$$ut\sin(\alpha-\beta) - \frac{1}{2}gt^2\cos\beta = 0$$

$$t\left(u\sin(\alpha-\beta) - \frac{1}{2}gt\cos\beta\right) = 0$$

$$t \neq 0, \quad u\sin(\alpha-\beta) - \frac{1}{2}gt\cos\beta = 0$$

$$\frac{1}{2}gt\cos\beta = u\sin(\alpha-\beta)$$

$$t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

To find the maximum Range on the inclined plane.

$$R = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$$

$$= \frac{u^2[2\cos\alpha\sin(\alpha-\beta)]}{g\cos^2\beta}$$

$$= \frac{u^2[\sin(2\alpha-\beta) + \sin(-\beta)]}{g\cos^2\beta}$$

$$= \frac{u^2[\sin(2\alpha-\beta) - \sin\beta]}{g(1-\sin^2\beta)}$$

R to hit a maximum Range

if $\sin(2\alpha-\beta) = 1$

$$2\alpha-\beta = \sin^{-1}(1)$$

$$2\alpha-\beta = \frac{\pi}{2} \Rightarrow 2\alpha = \frac{\pi}{2}+\beta \Rightarrow \boxed{\alpha = \frac{\pi}{4}+\frac{\beta}{2}}$$

$$R = \frac{u^2(1-\sin\beta)}{g(1+\sin\beta)(1-\sin\beta)}$$

$$\text{Maximum Range } (R) = \frac{u^2}{g(1+\sin\beta)}$$

To find Range, Time of flight and maximum Range on down the inclined plane.

Put $\beta = -\beta$

$$\text{Range } R = \frac{2u^2 \sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$$

$$\text{Time of flight, } T = \frac{2u\sin(\alpha+\beta)}{g\cos\beta}$$

$$\text{Maximum Range, } R = \frac{u^2}{g(1-\sin\beta)}$$

Problem

1. If R is maximum range on the inclined plane, T is their time of flight. P.T, $R = \frac{1}{2}gT^2$.

Sol

$$\text{Maximum range on inclined plane } R = \frac{u^2}{g(1+\sin\beta)}$$

$$\text{Time of flight } T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$\alpha = \frac{\pi}{4} + \frac{\beta}{2}$$

$$T = \frac{2u\sin\left(\frac{\pi}{4} + \beta/2 - \beta\right)}{g\cos\beta}$$

$$= \frac{2u\sin\left(\pi/4 - \beta/2\right)}{g\cos\beta}$$

$$= \frac{2u\left(\sin\pi/4 \cdot \cos\beta/2 - \cos\pi/4 \cdot \sin\beta/2\right)}{g\cos\beta}$$

$$= \frac{2u\left(\frac{1}{\sqrt{2}}\cos\beta/2 - \frac{1}{\sqrt{2}}\sin\beta/2\right)}{g\cos\beta}$$

$$= \frac{2u\cdot\frac{1}{\sqrt{2}}\left(\cos\frac{\beta}{2} - \sin\beta/2\right)}{g\cos\beta}$$

$$T = \frac{\sqrt{2}\,u\left(\cos\beta/2 - \sin\beta/2\right)}{g\cos\beta}$$

$$\frac{1}{2}gT^2 = \frac{1}{\cancel{2}}g\,\frac{\cancel{2}u^2\left(\cos\beta/2 - \sin\beta/2\right)^2}{g^2\cos^2\beta}$$

$$= \frac{u^2\left(\cos^2\beta/2 + \sin^2\beta/2 - 2\cos\beta/2\cdot\sin\beta/2\right)}{g^2(1-\sin^2\beta)}$$

$$= \frac{u^2\left(1 - 2\sin\beta/2\cdot\cos\beta/2\right)}{g^2(1-\sin^2\beta)}$$

$$= \frac{u^2\left(1 - \sin\beta\right)}{g^2(1-\sin^2\beta)}$$

$$= \frac{u^2\cancel{(1-\sin\beta)}}{g^2\cancel{(1-\sin\beta)}(1+\sin\beta)}$$

$$\frac{1}{2}gT^2 = \frac{u^2}{g^2(1+\sin\beta)}$$

$$\boxed{\frac{1}{2}gT^2 = R}$$

—x—

2) If the greatest range down on inclined plane is three times to the greatest range upon inclined plane. P.T the inclined plane at $\beta = 30^\circ$ to the horizontal.

Sol:

Greatest range down on inclined plane $\Big\}\ R_1 = \frac{u^2}{g(1-\sin\beta)}$

Greatest range up on inclined plane $\Big\}\ R_2 = \frac{u^2}{g(1+\sin\beta)}$

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Given $R_1 = 3R_2$

$$\frac{\cancel{u^2}}{\cancel{g}(1-\sin\beta)} = 3\cdot\frac{\cancel{u^2}}{\cancel{g}(1+\sin\beta)}$$

$$1+\sin\beta = 3\cdot(1-\sin\beta)$$

$$1+\sin\beta = 3-3\sin\beta$$

$$3\sin\beta+\sin\beta = 3-1$$

$$\cancel{4}^{\,2}\sin\beta = \cancel{2}$$

$$\sin\beta = \frac{1}{2}$$

$$\beta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

$$\boxed{\beta = 30^\circ}$$

---x---

3) A particle projected with the Speed 'u' Strikes at right angles a plane through the point of projection inclined at an angle β to the horizontal if α, T and R are angle of projection, Time of flight and Range on the inclined plane.

S.T.

i) $\cot\beta = 2\tan(\alpha-\beta)$

ii) $\cot\beta = \tan\alpha - 2\tan\beta$

iii) $T = \dfrac{2u}{g\sqrt{1+3\sin^2\beta}}$

(iv) $R = \dfrac{2u^2\sin\beta}{g(1+3\sin^2\beta)}$

Sol: i) Let u be the velocity of projection and α be the angle of projection

W.K.T Time of flight on inclined plane $t = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$ $\longrightarrow (1)$

But $v = u+at$

$$0 = u\cos(\alpha-\beta) - t_1 g\sin\beta$$

$$t_1 g\sin\beta = u\cos(\alpha-\beta)$$

$$t_1 = \frac{u\cos(\alpha-\beta)}{g\sin\beta} \longrightarrow (2)$$

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from (1) & (2)

$$t = t_1$$

$$\frac{2\cancel{u}\sin(\alpha-\beta)}{\cancel{u}\cos(\alpha-\beta)} = \frac{\cancel{g}\cos\beta}{\cancel{g}\sin\beta}$$

$$\boxed{2\tan(\alpha-\beta) = \cot\beta}$$

ii)

$$(i) \Rightarrow \cot\beta = 2\tan(\alpha-\beta)$$

$$\cot\beta = 2\left[\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}\right]$$

$$\cot\beta.(1+\tan\alpha\tan\beta) = 2(\tan\alpha - \tan\beta)$$

$$\cot\beta + \cancel{\cot\beta}.\tan\alpha.\cancel{\tan\beta} = 2\tan\alpha - 2\tan\beta$$

$$\cot\beta = 2\tan\alpha - 2\tan\beta - \tan\alpha$$

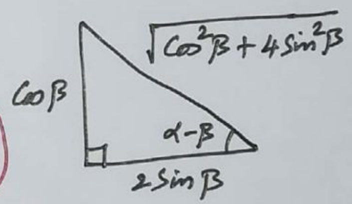
$$\boxed{\cot\beta = \tan\alpha - 2\tan\beta}$$

(iii)

$$(1) \Rightarrow \cot\beta = 2\tan(\alpha-\beta)$$

$$2\tan(\alpha-\beta) = \cot\beta$$

$$\tan(\alpha-\beta) = \frac{\cos\beta}{2\sin\beta} \left(= \frac{opp}{adj.}\right)$$



Time of flight $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta} \longrightarrow (*)$

$$\sin(\alpha-\beta) = \frac{\cos\beta}{\sqrt{\cos^2\beta + 4\sin^2\beta}} = \frac{\cos\beta}{\sqrt{1-\sin^2\beta+4\sin^2\beta}} \quad \left(= \frac{OPP}{Hyp}\right)$$

$$\sin(\alpha-\beta) = \frac{\cos\beta}{\sqrt{1+3\sin^2\beta}}$$

$$(*) \Rightarrow T = \frac{2u}{g\cancel{\cos\beta}} \cdot \frac{\cancel{\cos\beta}}{\sqrt{1+3\sin^2\beta}} \Rightarrow \boxed{T = \frac{2u}{g\sqrt{1+3\sin^2\beta}}}$$

IV) W.K.T the equation of Motion

$$S = ut + \frac{1}{2}at^2 \longrightarrow (3)$$

$S = R, \quad u = u\cos(\alpha-\beta), \quad a = -g\sin\beta$

from (2)

$$t = \frac{u\cos(\alpha-\beta)}{g\sin\beta}$$

$$(3) \Rightarrow R = u\cos(\alpha-\beta) \cdot \frac{u\cos(\alpha-\beta)}{g\sin\beta} - \frac{1}{2}g\sin\beta\left(\frac{u^2\cos^2(\alpha-\beta)}{g^2\sin^2\beta}\right)$$

$$R = \frac{u^2\cos^2(\alpha-\beta)}{g\sin\beta} - \frac{1}{2}\frac{u^2\cos^2(\alpha-\beta)}{g\sin\beta}$$

$$= \frac{u^2\cos^2(\alpha-\beta)}{g\sin\beta}\left(1-\frac{1}{2}\right)$$

$$R = \frac{u^2\cos^2(\alpha-\beta)}{2g\sin\beta} \longrightarrow (**)$$

Now, $$\cos(\alpha-\beta) = \frac{2\sin\beta}{\sqrt{\cos^2\beta + 4\sin^2\beta}}$$

$$= \frac{2\sin\beta}{\sqrt{1-\sin^2\beta+4\sin^2\beta}}$$

$$= \frac{2\sin\beta}{\sqrt{1+3\sin^2\beta}}$$

$$(**) \Rightarrow R = \frac{u^2}{2g\sin\beta}\cdot\left(\frac{2\sin\beta}{\sqrt{1+3\sin^2\beta}}\right)^2 = \frac{\cancel{4}\,u^2\sin^{\cancel{2}}\beta}{\cancel{2}g\,\cancel{\sin\beta}\,(1+3\sin^2\beta)}$$

$$\boxed{R = \frac{2u^2\sin\beta}{g(1+3\sin^2\beta)}}$$

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